Terahertz lens made out of natural stone

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compiled: November 20, 2013

Terahertz (THz) time-domain spectroscopy probes the optical properties of the naturally occurring solid aggregates of minerals, or stones, in THz frequency range. Refractive index and extinction coefficient measurement reveals that most natural stones including mudstone, sandstone, granite, tuff, gneiss, diorite, slate, marble, and dolomite are fairly transparent for THz frequency waves. Dolomite in particular exhibits nearly uniform refractive index of 2.7 over the broad frequency range from 0.1 to 1 THz. The high index of refraction allows flexibility in lens designing with a shorter accessible focal length or a thinner lens with a given focal length. The good agreement between the experiment and calculation in the THz beam profile confirms that dolomite has high homogeneity as a lens material, suggesting the possibility of using natural stones for THz optical elements.

OCIS codes: (080.3630) Lenses; (300.6495) Spectroscopy, terahertz; (220.4610) Optical fabrication
http://dx.doi.org/10.1364/XX.99.099999

1. Introduction
Science and technology involved with THz frequency waves has become one of the most active areas of research during the past two decades [1]. The THz frequency waves are located in the frequency range 0.1–10 THz (30–3000 µm in wavelength) between microwave and far-infrared (FIR) electromagnetic waves. Various research in THz frequency range have become enabled through developing the generation and detection methods as well as spectroscopic methods with these waves. One of the important aspects of THz waves in material characterization application, in constrast to the measurements with other light sources such as X-ray or FIR waves, is the direct field amplitude measurement in THz time-domain spectroscopy (THz-TDS). Both the amplitude and phase information of a sample can be obtained simultaneously through the THz-TDS, without resorting to a complex analysis [2]. Previous studies demonstrated the powerful spectroscopic capability of THz-TDS in characterizing many materials including polymers [3], explosive materials [4], solid-state materials [5, 6], chemical compounds in liquid [7], ion [8], biomaterials [9], and even the material phase transitions [10].

Recently there have been enormous efforts devoted to the fabrication of THz optical components [11–18]. Among the THz components, lenses and off-axis parabolic mirrors play a crucial role in THz-TDS systems, which are the basic optical elements in focusing and collimating THz waves. For example, planoconvex lenses are commonly used in a THz-TDS system of a linear configuration [11]. The lenses operating in the THz frequency range are fabricated with various materials: high resistive silicon [12, 13], polytetrafluoroethylene (PTFE, Teflon) [3, 12–14], high-density polyethylene (HDPE) [3, 11, 14], TPX [13], TOPAS [13, 14], Zeonex [15], Picarin [16], micropowders [17], and polymeric compounds [18]. Also, special lenses such as the off-axis metallic diffractive lens [19], the diffractive paper lens [20], the variable-focus lens using the medical white oil [21], and the THz Brewster lens [22] have been also recently reported.

In this paper, we demonstrate a THz lens made out of natural dolomite stone. For this, we first investigate various natural stones using THz-TDS to determine their optical constants such as the refraction index and the absorption coefficient. The optical constants of a limited set of stones were previously studied in THz frequency range [23–25], but most stones from nature are yet to be studied. In this study, we investigate mudstone, sandstone, tuff, diorite, marble, granite, gneiss, slate, and dolomite. While these stones are opaque in optical frequency range, the result in this study reveals that they are mostly transparent in frequency range from 0.2 to 1 THz. Some natural stones, in particular the dolomite, exhibit a rather flat and high index of refraction throughout the measured THz frequency range. Note that materials with a high refractive index allow the lens fabrication with more flexibility suggesting that the dolomite can be considered as a material for THz lens fabrication.

The contents in this paper are listed as follows. After we briefly explain the THz transmission measurement of natural stones in a conventional THz-TDS setup, we describe the extraction process of the complex refractive index from the investigated stones. The fabrication pro-
2. Measurement of optical constants of natural stones

2.A. Experimental procedure

For the experimental measurement of the refractive index and the extinction coefficient of various natural stones, we used a conventional THz-TDS setup [6]. THz waves were generated from a large-area photoconductive antenna (PCA) [26] illuminated by ultrafast optical pulses that were temporally 100 fs short, wavelength-centered at 840 nm, and produced from a mode-locked TiSapphire laser oscillator operating at 80 MHz repetition rate. When a THz pulse was guided by a four off-axis parabolic mirrors through the sample located at the focus, another ultrafast optical pulse that was split off before the PCA and time-delayed by a linear translation stage probed the electric field profile of the THz wave as a function of the time delay via optical gating [13]. The temporal THz signals with and without the sample were separately measured via electro-optic (EO) sampling [27, 28], where the polarization rotation of the probe pulse through a (110)-oriented ZnTe EO crystal with thickness of 2 mm was mapped after a quarter-wave plate and a Wollaston prism by a pair of balanced photodiodes. By varying the time-delay of the probe beam with respect to the THz pulse, the temporal electric field waveform of the THz pulse was recorded. To obtain an accurate spectral information, we took a long-time window measurement of up to 200 ps, which corresponded to the spectral resolution of 5 GHz, with a temporal step size of 100 fs. The whole THz-TDS setup was purged with dry air to reduce the absorption by water vapor in THz frequency range [29, 30].

2.B. Retrieval of refractive index and extinction coefficient

The waveform measurement in THz-TDS allows one to obtain not only the spectral amplitude but also the spectral phase information by simply applying the Fourier transformation to the time domain signal. The THz-TDS directly measures the electric field, while conventional IR spectroscopy such as FT-IR (Fourier transform IR spectroscopy) measures intensity [31], so the spectral phase information is obtained without resorting to Kramers-Kronig relationship. The transmitted THz electric field out from the sample is given as a sum of successive transmitted and reflected electric fields at both sides of the sample, which is often referred to as Fabry-Pérot etalon signal. If we denote air and the sample by subscripts 1 and 2, respectively, then the transmission \( T(\omega) \) of the sample with thickness \( t \), which is separately measured in the experiment, is given by

\[
T(\omega) = t_{12} t_{12} e^{i \frac{\pi}{2} (n_s - n_a) t} \left\{ 1 + \sum_{j=1}^{S} \left[ r_{21} e^{i \pi n_s (\omega)} \right]^{2j} \right\} = \rho(\omega) e^{i \Delta \phi(\omega)} \left\{ FP(\omega) \right\},
\]

(1)

where \( \tilde{n} = n_s + i \kappa_s \) and \( n_a \) are the complex indices of refraction of the sample and the air, respectively, \( \delta \) is the number of echoes of a THz signal, \( \rho \) is the transmission amplitude, and \( \Delta \phi \) is the spectral phase difference between the THz signals with and without the sample, or simply the transmission phase. Also, \( t_{ij} \) and \( r_{ij} \) are the Fresnel coefficients given by \( t_{ij} = 2 n_i / (\tilde{n}_i + \tilde{n}_j) \), \( r_{ij} = (\tilde{n}_j - \tilde{n}_i) / (\tilde{n}_j + \tilde{n}_i) \), and \( FP(\omega) \) is the Fabry-Pérot term. By comparing the real and imaginary parts of the right-hand side of Eq. (1), the full expression for the complex index of refraction of the sample becomes

\[
n_s = n_a + \frac{c}{\omega} \left( \Delta \phi - \tan^{-1} \frac{\kappa_s}{n_a} + 2 \tan^{-1} \frac{\kappa_s}{n_s + n_a} \right).
\]

(2)

\[
\kappa_s = \frac{c}{\omega} \left( \log^2 \frac{4 n_a \sqrt{n_s^2 + \kappa_s^2}}{(n_s + n_a)^2 + \kappa_s^2} - \log \rho \right).
\]

(3)

For the retrieval of refractive index and extinction coefficient, we used the fixed-point iteration method [32, 33], in which the initial values of the complex index of refraction was chosen in the fixed-point iteration as
The index of refraction of dolomite is generally lower than that of other materials, such as Teflon, which is commonly used in THz technology. Dolomite has a higher refractive index (2.70) compared to Teflon (1.42) and other materials tested, with an extinction coefficient of about 2 cm⁻¹. This makes dolomite an attractive material for THz applications, especially for lenses, which require a balance between high refractive index and low absorption.

Dolomite is particularly useful in the THz frequency range (0.2–1.2 THz), where it shows low absorption and nearly uniform refractive index. The high refractive index and low extinction coefficient of dolomite enable the fabrication of planoconvex lenses, as demonstrated in the figure. These lenses have a focal length of 9.4 mm and a flat lateral size of 2×2 cm², with thicknesses ranging from 6.8 to 9.4 mm. The lenses are fabricated using a hexahedron-shaped block of dolomite, which is cut into a planoconvex shape using a diamond wheel cutter. The lens shape is then polished on a rotating plate to achieve the desired curvature.

Table 1 lists the refractive indices and extinction coefficients for various natural stones, along with typical lens materials. Dolomite has a higher refractive index (2.70) and lower extinction coefficient (4.1) compared to Teflon (1.42) and other materials, making it a promising material for THz applications.

Table 2 shows the refractive indices and extinction coefficients of dolomite and typical lens materials measured at 0.5 THz. Dolomite has a higher refractive index and lower extinction coefficient compared to other materials, making it an excellent choice for THz lenses.

Equation (3) shows how to calculate the extinction coefficient from the imaginary part of the complex refractive index:

\[ \alpha(\omega) = \frac{2\omega}{c}\kappa_s(\omega). \]

In this equation, \( \omega \) is the angular frequency, \( c \) is the speed of light, and \( \kappa_s(\omega) \) is the extinction coefficient at a given frequency. This equation is used to determine the absorption properties of materials in the THz spectrum.
until the painted surface vanishes. And then, SiC powders of 600 and 1200 mesh numbers were sequentially used to smoothly grind the surfaces. After the grinding process, a polishing film (3M 261X Imperial Lapping film, 3 µm grade) doped with aluminium oxide (Al₂O₃) was employed to roughly polish the smoothed surface of the dolomite. Then, fine polishing procedure with cerium oxide (CeO₂) abrasive composed of 1 µm-size powder followed.

After the fine polishing, the expected surface quality of our dolomite lens in THz frequency range is over λ/1000. The radius of curvature of our dolomite lens is 16.19 cm which is the same to the one of the polishing plate, and the thickness of the lens at the center, t₀, and at the edge, tₑ, are measured to be 4.1 mm, and 2.1 mm, respectively, and the difference between t₀ and tₑ is the calculated value (1.958 mm) concerning the radius of the lens. So, the expected focal length, fₑ is estimated to be 95.2 mm, as previously mentioned, and the back focal length, fₑ, is 93.1 mm. All the parameters of the lens are graphically shown in Fig. 2–(a) along with the actual photo image of the fabricated lens in in Fig. 2–(b).

![THz-TDS Setup](image)

Fig. 3. (a) Schematic of our linear configuration THz-TDS setup using two THz lenses: a Teflon lens and the fabricated dolomite lens. The overall intensity profiles were measured by moving the lens with an XYZ translation stage. (b) Photo of the THz-TDS setup corresponds to the boxed area of (a). The THz field was focused on to ZnTe by the fabricated dolomite lens, and ITO, playing a role as a dichroic polarization beam splitter reflected the THz field and transmits the probe beam. (c) Temporal THz amplitude signal measured at the focal point and the corresponding amplitude spectrum after Fourier transformation.

### 4. Results and Discussion

The shape of the focused THz field was examined by the field profile measurement with a THz-TDS setup in a linear configuration as depicted in Fig. 3–(a), and the photo of the setup is shown in Fig. 3–(b). In this configuration, the diverging THz wave generated from the PCA was collimated by a Teflon lens with a focal length of 10 cm, and focused by the dolomite lens. Then, the THz pulse and the probe laser pulse were merged by an ITO wafer, and THz signal was detected by the ZnTe crystal. We determined the focal point of the dolomite lens to be the maximum point of the measured THz signal by scanning the spatial amplitude with an XYZ translation stage. The field profile of the dolomite lens was obtained by a two-dimensional (2D) areal scanning of 9 × 9 cm² with an interval of 300 µm. At each point of the measured 2D area, the temporal profile measurement over 10 ps time window was carried out. The maximum THz signal in the center pixel of the 2D area and the corresponding spectral amplitude obtained by the Fourier transform are shown in Figs. 3–(c).

The experimental measurement geometry of the beam profile in Fig. 4–(a) is theoretically equivalent to the measurement of the Fraunhofer diffraction pattern from a circular aperture. The dolomite lens and Teflon lens have same diameter of 2 inches. The second lens, here the dolomite lens, yields the size of the aperture in the Fraunhofer diffraction. This lens crops the THz waves reaching the lens and only a circular segment propagates through the lens and forms the diffraction pattern in the focal plane [36]. The THz pulse in our experimental condition has a broadband spectrum and therefore the performance of the dolomite lens can be confirmed by analyzing the beam profiles for various frequencies. We fit our amplitude profile $E(r)$, where $r$ is the radial distance on the focal plane, to the Bessel pattern [36] given by

$$E(r) = E(0) \left| \frac{2 J_1(kW₀r/2f)}{kW₀r/2f} \right|,$$

where $J_1$ is the first kind Bessel function of order one, k is the wavenumber, f (95.2 mm) is the focal length of the lens, and $W₀$ is the diameter of the lens.

Figure 4–(b) summarizes the extracted diameters (FWHM) of the focused THz field as a function of wavelength obtained from the numerical fit of the amplitude profiles to the Bessel pattern in Eq. (5). The measured THz beam amplitude profiles at various frequencies, the corresponding x-, and y-cross sections of the profiles are shown in Fig. 4–(c). The amplitude profiles were scaled by the THz signal amplitude without the dolomite lens, and the corresponding ratio, $N_{ij}$, is given by

$$N_{ij}(ω_k) = \frac{|E_{ij}(ω_k)|}{|E_{no\,lens}(ω_k)|},$$

where $|E_{ij}(ω_k)|$ and $|E_{no\,lens}(ω_k)|$ are the amplitudes of THz signals with and without the fabricated dolomite lens, respectively. The amplitude attenuation increases
as a function of the frequency shown in Fig. 4–(c). Nevertheless, the experimental result agrees well with the calculation of the THz beam profile. This result verifies that dolomite has high homogeneity as a lens material although dolomite has a few more absorption than other lens materials.

As expected, the beam diameters show the decreasing behavior as the frequency increases, and are compared with the theoretical line calculated with the focal length of $f = 95.2$ mm and the lens diameter of $W_0 = 50$ mm (maximal $f$-number = 1.9). The theoretical line is calculated FWHM, $W(\lambda)$, from the Bessel pattern in Eq. (5) as

$$W(\lambda) = \frac{4.43 f}{\pi W_0} \lambda.$$  

The good agreement between the experiment and calculation confirms again that the fabricated lens made out of dolomite shows good performance in THz frequency range.

5. Conclusion

In summary, we described the use of natural stones as an optical element material in THz frequency range. For this, we measured optical constants of various stones from nature using THz-TDS, and, among the investigated stones, dolomite in particular exhibited the flat refractive index and low absorption over the measured THz frequency range. We fabricated a dolomite planoconvex lens using the conventional lens making procedure. The measured focused beam profiles was...
well explained by far-field diffraction theory in THz frequency range. With the proof-of-principle demonstration of THz lens made out of dolomite, we suggest the possibility of using natural stones for THz optical elements from scientific and economic aspects.

Acknowledgments
This research was supported in part by Basic Science Research Programs [2013R1A2A2A05005187, 2009-0093428] and in part by the WCI Program [WCI 2011-001] through the National Research Foundation of Korea.

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